ADJOINT PERTURBATION THEORY FOR FLUXES AND RADIANCES

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Introduction

- The perturbation technique is one of the most important and unique tools at hand. It allows
  - to reduce the complicatedness of a given problem,
  - to decrease the computational load.

- The favorite application of the perturbation technique is to estimate how significant the deviation of a given problem from the one can be solved employing a simple technique.

- It is a general knowledge that simulation of radiative transfer through a realistic 3D cloud requires a lot of computer power and time. That is why any simplification of the simulation scheme is of great interest, especially, for remote sensing and cloud modeling applications. In this case the perturbation technique has advantage enabling one to estimate the effect of an **arbitrary** cloud structure on the radiance propagation using close form expressions.

- There are several ways to formulate the perturbation approach, but here the one based on variational principle will be considered.

Variational principle to derive RTE

$$W = \int_{\Xi} \tilde{I} \left[\vec{n} \cdot \vec{\nabla} I + \sigma_e I - \frac{\sigma_s}{4\pi} \int_{4\pi} P(\vec{n}, \vec{n}') I(\vec{r}, \vec{n}') d\vec{n}' \right] d\Xi -$$

$$\int_{\Xi} \left[\tilde{I} S + R I \right] d\Xi + \int_{\mu > 0} \mu \left[\tilde{I} I - \tilde{I} \int_{\mu < 0} A(\vec{r}, \vec{n}, \vec{n}') I(\vec{r}, \vec{n}') \mu' d\vec{n}' \right]_{\vec{r} \in \Sigma} d\vec{n}$$

Ξ is the problem parameter space, $\tilde{I}$ is the solution of the adjoint RTE, I is the solution of the direct RTE, σ_e and σ_s are the extinction and scattering coefficients, respectively, $P(\vec{n}, \vec{n}')$ is the phase function, S and R are the source and receiver functions, and $A(\vec{r}, \vec{n}, \vec{n}')$ describes refelection properties of the boundary surface.

RTE's

Direct

$$\vec{n}\cdot\vec{\nabla}I-\sigma_e I-\frac{1}{4\pi}\int_{4\pi}P(\vec{n},\vec{n}')I(\vec{r},\vec{n}')d\vec{n}'-S=0,$$

$$I=\int_{\mu<0}A(\vec{r},\vec{n},\vec{n}')I(\vec{r},\vec{n}')\mu' d\vec{n}',\ \text{for } \vec{r}\in\Sigma$$

Adjoint

$$-\vec{n}\cdot\vec{\nabla}\tilde{I}-\sigma_e\tilde{I}-\frac{1}{4\pi}\int_{4\pi}P(-\vec{n},-\vec{n}')\tilde{I}(\vec{r},\vec{n}')d\vec{n}'-R=0,$$

$$\tilde{I}=\int_{\mu>0}A(\vec{r},-\vec{n},-\vec{n}')\tilde{I}(\vec{r},\vec{n}')\mu' d\vec{n}',\ \text{for } \vec{r}\in\Sigma$$

At the point of minimum the functional value

$$P=-W=\int_{\Xi}\tilde{I}S\,d\Xi=\int_{\Xi}IR\,d\Xi.$$

Perturbation technique

Variation of the functional at the point of minimum

Medium parameter variation

$$\Delta E = -\Delta W = \int_{\Xi} \tilde{I} \left[-\Delta\sigma_e I + \frac{1}{4\pi} \int_{4\pi} \left(\Delta\sigma_s P(\vec{n},\vec{n}') + \sigma_s \Delta P(\vec{n},\vec{n}') \right) I(\vec{r},\vec{n}') d\vec{n}' \right] d\Xi +$$

$$+ \int_{\Xi} \left[\tilde{I} \Delta S + \Delta R I \right] d\Xi + \int_{\mu>0} \mu \left[\tilde{I} \int_{\mu<0} \Delta A(\vec{r},\vec{n},\vec{n}') I(\vec{r},\vec{n}') \mu' d\vec{n}' \right]_{\vec{r}\in\Sigma} d\vec{n}$$

Source and Receiver variation

Boundary condition variation

Perturbation technique summary

- **Advantages:**
  - a variation of any cloud field optical properties;
  - any radiance characteristics;
  - possibility of close form expressions;
  - analytical insights.

- **Drawbacks:**
  - it is difficult to obtain an *a priori* estimation of the accuracy;
  - unclear the variation strength limits.

Orthodox perturbation

Why conventional? To solve 3D problem we start from some average effective medium, and then consider the difference between real and effective as the perturbation

$$\bar{\sigma}_e = \frac{\int \sigma_e dxdy}{\int dxdy}$$

Quantity of interest is the radiance density

$$U(\vec{r}) = \frac{1}{4\pi} \int_{4\pi} I(\vec{r}, \vec{n}) d\vec{n}$$

A toy problem

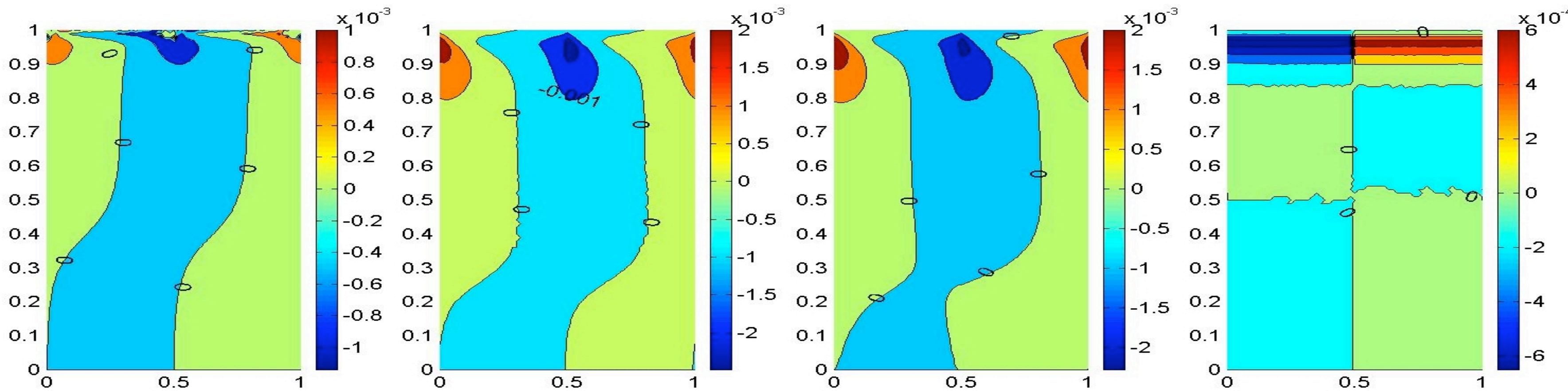
- We calculate the radiance density within a cloud with a square wave extinction coefficient variation:

$$\sigma_e = \begin{cases} \bar{\sigma}_e + \Delta\sigma_e, & 2Ln < x < L + 2Ln \\ \bar{\sigma}_e - \Delta\sigma_e, & 2Ln + L < x < 2L(n+1) \end{cases} = \bar{\sigma}_e + \sum_n \frac{4\Delta\sigma_e}{\pi(2n+1)} \sin\left[\frac{\pi(2n+1)}{L} x\right]$$

- H=1 km, L=0.5 km, and average σ_e=64 km^{-1}
- The phase function and single scattering albedo assume to be the same within the cloud: ω_0=1.0, g=0.85.
- In this case, the variation of the radiance density variation has the form

$$E(z,x) - \bar{E}(z) = \sum_n \Delta\sigma_e \frac{4\rho(z,k_n)}{\pi(2n+1)} \sin\left[k_n x + \phi(z,k_n)\right], \qquad k_n = \frac{\pi(2n+1)}{L}$$

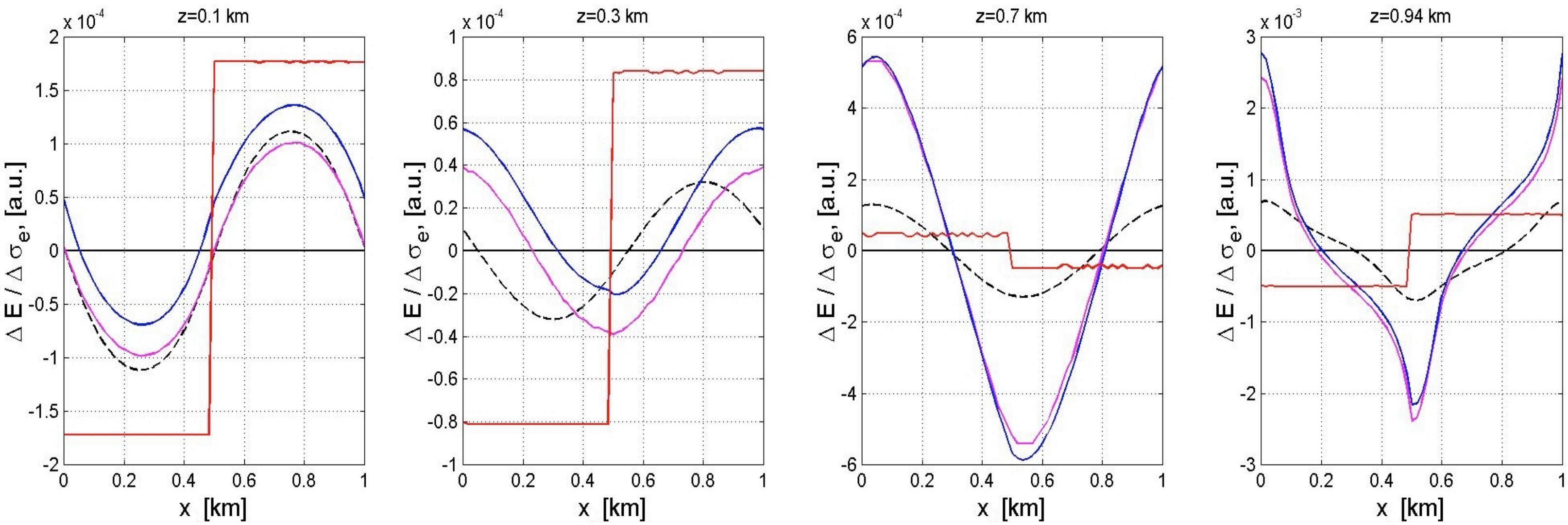
SHDOM, ICA, and Perturbation. I



Diffusion perturbation **$\Delta\sigma_e$ =1 km^{-1}** **$\Delta\sigma_e$ =16 km^{-1}** **independent pixel ($\Delta\sigma_e$ =1 km^{-1})**

Variation of the radiance density radiance normalized on $\Delta\sigma_e$. The Sun cosine is 0.5. Note the shifted position of the isolines with respect to square wave oscillations.

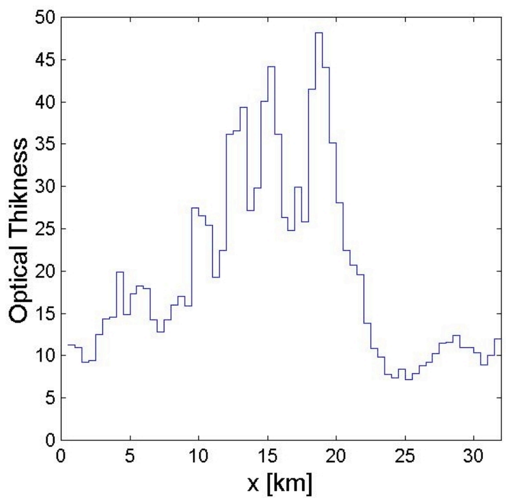
SHDOM, ICA, and Perturbation. II



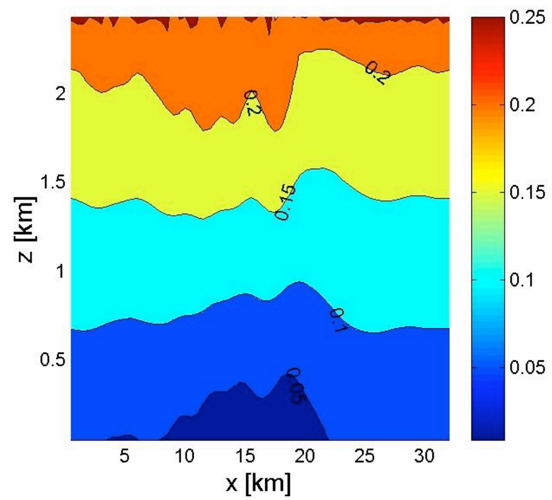
Selected cross section show that the perturbation (black dashed lines) provides a more accurate estimation than the ICA (red lines). The SHDOM results are depicted by the magenta ($\Delta\sigma e$ **=**1 km^{-1}) and blue ($\Delta\sigma e$ =16 km^{-1}) lines.

I3RC cloud with vertical homogeneity

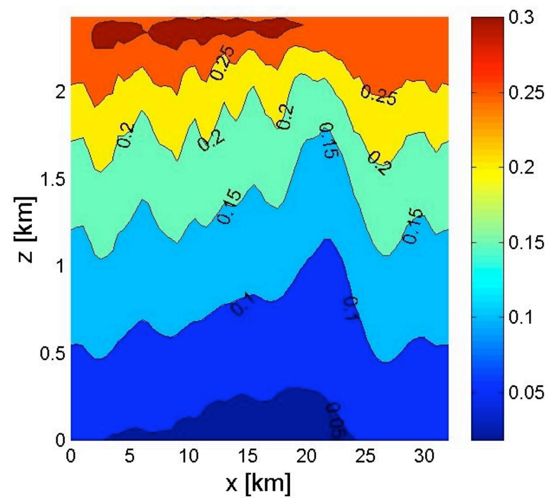
The optical thickness profile is shown below. The cloud is 2-dimensional with H=2.3 km and X=32 km. The Sun cosine is 0.5. The <τ>=19.6.



Optical thickness profile



Perturbation



SHDOM

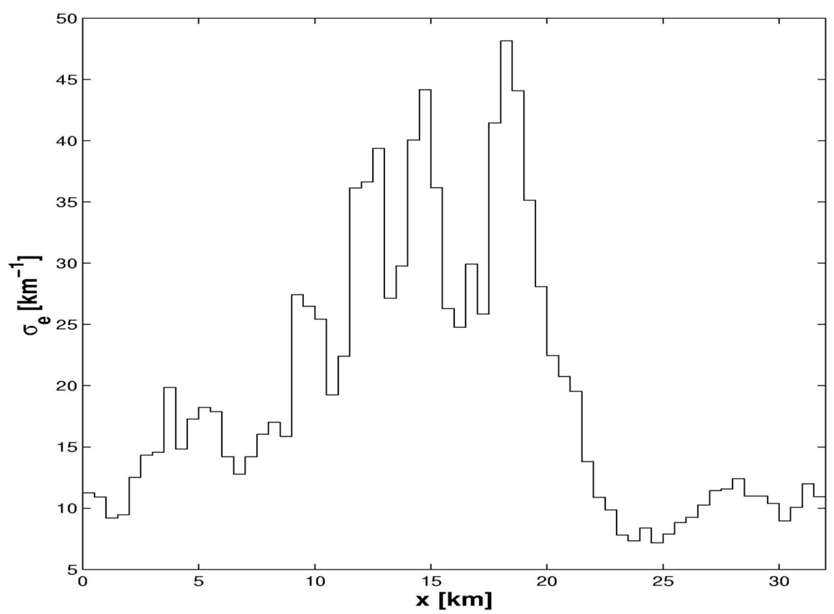
Non standard perturbation

Let us consider a part of the differential operator of the RTE as perturbation

$$\Delta L = \sin(\theta)\cos(\phi)\frac{d}{dx} + \sin(\theta)\sin(\phi)\frac{d}{dy}$$

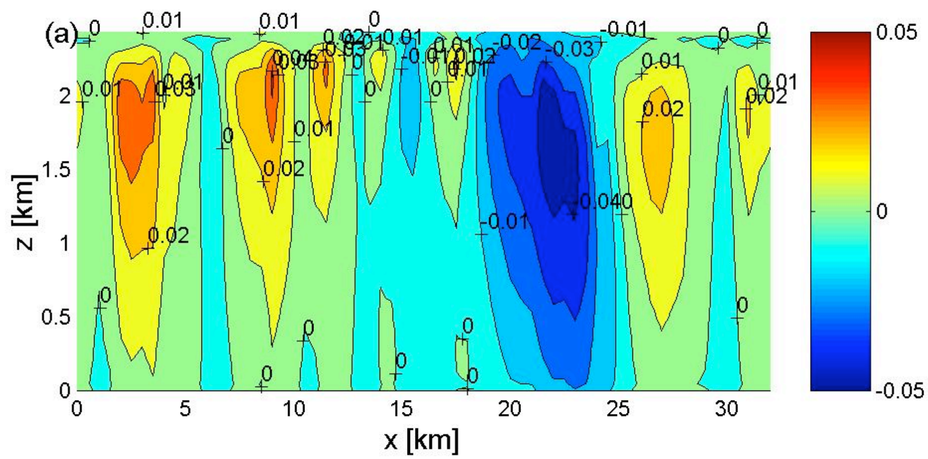
Now our base case is a true 1D problem which we shall solve using the diffusion approximation.

For simulation we shall use the same I3RC cloud model.

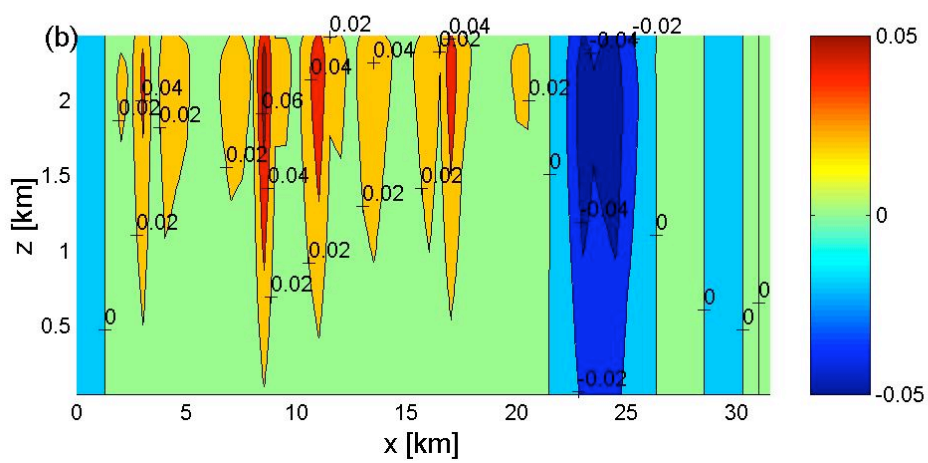


SHDOM vs Perturbation

Correction to the ICA calculated using the perturbation approximation and an accurate numerical simulation (SHDOM) .



- Perturbation approach lacks smoothness
- Perturbation corrections are more significant near cloud top and tends to zero at the bottom.



Orthodox vs Non-standard

- Orthodox approach
  - any angle of incidence;
  - small variations of the medium properties

- Non-standard
  - allows one to estimate only the adjacency effect due to slant illumination
  - applicable at any variations of the medium properties

Conclusions

- The comparison of the *diffusion perturbation technique* predictions with the results of the SHDOM simulation shows its good accuracy to depict the radiance density (or the solar heating rate).

- Incorporation of a fully analytical diffusion perturbation is simple and allows one to obtain a significant improvement with respect to the ICA.